6.1 Polygons

Goals
- Identify, name, and describe polygons.
- Use the sum of the measures of the interior angles of a quadrilateral.

VOCABULARY

Polygon A polygon is a plane figure that meets the following conditions: (1) It is formed by three or more segments called sides, such that no two sides with a common endpoint are collinear. (2) Each side intersects exactly two other sides, one at each endpoint.

Sides The sides of a polygon are the segments that form the polygon.

Vertex A vertex of a polygon is an endpoint of a side of the polygon.

Convex A polygon is convex if no line that contains a side of the polygon contains a point in the interior of the polygon.

Nonconvex A nonconvex polygon is a polygon that is not convex.

Concave A concave polygon is a polygon that is not convex.

Equilateral A polygon is equilateral if all of its sides are congruent.

Equiangular A polygon is equiangular if all of its interior angles are congruent.

Regular A polygon is regular if it is equilateral and equiangular.

Diagonal A diagonal of a polygon is a segment that joins two nonconsecutive vertices.
Example 1  
*Identifying Polygons*

State whether the figure is a polygon. If it is not, explain why.

Solution

Figures  \( C \) and  \( D \) are polygons.

- Figure  \( A \) is not a polygon because \textit{one of its sides intersects more than two other sides}.
- Figure  \( B \) is not a polygon because \textit{some of its sides are not segments}.
- Figure  \( E \) is not a polygon because \textit{two of the sides intersect only one other side}.

Example 2  
*Identifying Convex and Concave Polygons*

Identify the polygon and state whether it is convex or concave.

a.  

b.

Solution

a. The polygon has 6 sides, so it is a \textit{hexagon}. When extended, none of the sides intersect the interior, so the polygon is \textit{convex}.

b. The polygon has 7 sides, so it is a \textit{heptagon}. When extended, some of the sides intersect the interior, so the polygon is \textit{concave}.
Example 3 \hspace{1cm} Identifying Regular Polygons

Decide whether the polygon is regular.

a. \hspace{1cm} b. \hspace{1cm} c.

\begin{align*}
\text{Solution} \\
\text{a. The polygon is} \, \text{equilateral, but it is not} \, \text{equiangular. So, it} \, \text{is not} \, \text{regular.} \\
\text{b. The polygon is} \, \text{equiangular, but it is not} \, \text{equilateral. So, it} \, \text{is not} \, \text{regular.} \\
\text{c. The polygon is} \, \text{equilateral and} \, \text{equiangular. So, it} \, \text{is regular.}
\end{align*}

\checkmark \hspace{0.5cm} \textbf{Checkpoint} \hspace{0.5cm} \text{Name the polygon. Is the polygon convex or concave? Is it regular?}

\begin{align*}
\text{1. quadrilateral; convex; no} \\
\text{2. pentagon; concave; no} \\
\text{3. hexagon; convex; yes}
\end{align*}

\textbf{THEOREM 6.1: INTERIOR ANGLES OF A QUADRILATERAL}

The sum of the measures of the interior angles of a quadrilateral is \(360^\circ\).

\[ m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ \]
Example 4  Interior Angles of a Quadrilateral

Find $m\angle U$ and $m\angle V$.

Solution

Find the value of $x$. Use the sum of the measures of the interior angles to write an equation involving $x$. Then, solve the equation.

$$5x^\circ + (3x + 10)^\circ + 72^\circ + 118^\circ = 360^\circ$$

$$8x + 200 = 360$$

$$8x = 160$$

$$x = 20$$

Find $m\angle U$ and $m\angle V$.

$$m\angle U = 5x^\circ = (5 \cdot 20)^\circ = 100^\circ$$

$$m\angle V = (3x + 10)^\circ = (3 \cdot 20 + 10)^\circ = 70^\circ$$

Answer So, $m\angle U = 100^\circ$ and $m\angle V = 70^\circ$.

Checkpoint  Find $m\angle D$.

4.

5.